Nonlinear and spin effects in two-photon annihilation of a fermion pair in an intensive laser wave

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Abstract

The pattern of calculation of amplitudes of a series of processes in the field of an intensive laser wave, in which two fermions $(p; p')$ and two real photons $(k_1; k_2)$ participate, is considered. In relation to one-photon processes, these processes are of the second order on α, if the wave intensity $\xi \ll 1$ (i.e., actually absorption from the wave only one quantum). Otherwise, they are competing and essentially nonlinear. One-photon processes have a number of the important physical applications. For example, γe and $\gamma\gamma$ colliders work on their basis.

In DSB the calculation is conducted at the level of reaction amplitudes. It essentially simplifies both the calculation and the form of obtained results; those combinations of amplitudes which describe the spin effects are easy to calculate. And these effects are especially essential in nonlinear processes. The calculations are conducted in covariant form. Besides compactness, this provides independence of the frames of reference and energy modes. The masses of fermions are taken into account precisely and are not assumed equal each other (heavy leptons, modes of various decays). The pattern of calculation is given in the channel indicated in the title. The amplitudes of this channel are also applicable for calculating of induced decay of orthopositronium into two photons. And in this approach there is no necessity to assume the relative momentum equal to zero.

Keywords: the calculations of reaction amplitudes, the processes in external fields, orthopositronium, γ-colliders.

The process under consideration is described by 2^5 amplitudes. Because of parity conservation, we need to calculate 16 amplitudes (in our approach, they are described by 8 formulas). Therefore, here we can present only the pattern of calculation. The necessary formulas are cited in accordance with monograph [1] with the indication paragraph number,where §40 "An electron in the field of the plane electromagnetic wave"; §89 "An annihilation of a positronium"; §101 "Radiation of a photon by an electron in the field of an intensive electromagnetic wave". Unlike [1], we assume $\gamma_5 = i\gamma^0\gamma^1\gamma^2\gamma^3$ (opposite sign) and elementary charge $e > 0$.

The calculation of amplitudes will be made in the Diagonal Spin Basis (DSB) introduced by the author in [5]. A review of the method and the calculation of number processes analogous to that under consideration (but with participation only one photon) is given in [6].

In any reaction, an even number of fermions participate. Therefore, the amplitude of each reaction can be represented as a combination of fermion "sandwiches" describing the fermion line

$$\bar{u}^{\sigma'}(p')Qu^{\sigma}(p) = Tr[Qu^{\sigma}(p)\bar{u}^{\sigma'}(p')] \ , \tag{1}$$

where Q is the operator of interaction. If it has the tensor indices, there occurs contraction over them, with other "sandwiches" of other fermion lines. In the DSB, the axes of the spin projections belonging to one fermion line are lying in the 2-plane ($v = p/m,\ v' = p'/m'$). Vectors v', s_3' and bispinor $u^{\delta}(p')$ are obtained from v, s_3 and $u^{\delta}(p)$ by one and the same plane Lorentz transformation.

$$\begin{aligned}
&s_3 = \frac{(vv')v - v'}{\sqrt{(vv')^2-1}} \ , \quad s_3' = -\frac{(vv')v' - v}{\sqrt{(vv')^2-1}} \ ; \\
&s_1 = s_1' = n_1 = (\tilde{g} - g)\frac{k}{k_\perp} \ , \quad s_2 = s_2' = n_2 = -\tilde{\varepsilon}\frac{k}{k_\perp} \ ; \\
&n_0 = \frac{v+v'}{\sqrt{2(vv'+1)}} \ , \ n_3 = \frac{v-v'}{\sqrt{2(vv'-1)}} \ ; \ \tilde{g}^{\mu\nu} = n_0^\mu n_0^\nu - n_3^\mu n_3^\nu \ , \ \tilde{\varepsilon}^{\mu\nu} = \varepsilon^{\mu\nu}{}_{\rho\sigma} n_0^\rho n_3^\sigma \ .
\end{aligned} \tag{2}$$

The bispinors of both particles have a common set of spin operators and are related by the relation $u^{\delta}(p') = \hat{n}_0 u^{\delta}(p)$, $\delta = \pm 1$. If the initial or the final (our case) particle is an antifermion, this corresponds to the creation or annihilation of a pair. In this case, $v^{\delta}(p') = \delta\gamma_5 u^{-\delta}(p')$. In the DSB, the description of the spin and all the calculations made are covariant. In the case of annihilation, the spin indices in bispinors have the meaning of helicity in the system of centre of masses.

By virtue of the above properties, the fermion line in DSB acts as a certain single object, and the basic formulas for calculating the amplitudes (transition operators) acquire the elementary form:

$$u^{\delta}(p)\bar{v}^{\delta}(p') = \frac{\gamma_5}{4}\left[(V^+ + \delta\gamma_5 V^-)(1 - \delta\gamma_5\hat{n}_0\hat{n}_3) - \hat{n}_0 - \delta\gamma_5\hat{n}_3\right] \ , \tag{3}$$

$$u^{\delta}(p)\bar{v}^{-\delta}(p') = \frac{1}{4}(V^+ + \delta\gamma_5 V^- + \hat{n}_0)(\hat{n}_1 + i\delta\hat{n}_2) \ , \tag{4}$$

where $V^{\pm} = \sqrt{(vv' \pm 1)/2}$. As vector k, which enters into the definition of n_1 and n_2, we take the momentum of laser photon. The replacement of this vector by another leads to remultiplication of Eq.4 by the easily calculated phase factor.

If in tetrad n_A Eq.2 for fermion line we make the replacement $p \to k_1$, $p' \to k_2$, then we obtain tetrad h_A common for both photons. In this case, the vector $e_\lambda = \frac{1}{\sqrt{2}}(h_1 - i\lambda h_2)$ simultaneously describes both the emitted photons: k_1 with helicity $\lambda_1 = \lambda$ and k_2 with $\lambda_2 = -\lambda$. Thus, if $\lambda_1 + \lambda_2 = 0$, then only one operator $\hat{e}_\lambda$ enters into the amplitude, if $\lambda_1 + \lambda_2 = \pm 2$, then $\hat{e}_\lambda$ and $\hat{e}^*_\lambda$ enter into the amplitude. This fact also extremely simplifies both the structure and the calculation of the reaction amplitudes. Thus,

$$\begin{aligned} &\hat{e}^2_\lambda = 0 \ , \ \ \hat{e}_\lambda \hat{e}^*_\lambda = -1 + \frac{\gamma_5}{k_1 k_2} k_1^\mu \sigma_{\mu\nu} k_2^\nu \ ; \\ &\hat{e}_\lambda \hat{f} \hat{e}_\lambda = 2 e_\lambda f \hat{e}_\lambda \ ; \ \ \hat{e}_\lambda \hat{f} \hat{e}^*_\lambda = \frac{2}{k_1 k_2}(f k_2 \omega_{-\lambda} \hat{k}_1 + f k_1 \omega_\lambda \hat{k}_2) \ . \end{aligned} \tag{5}$$

where f is arbitrary vector, $\omega_\lambda = \frac{1}{2}(1 + \lambda\gamma_5)$.

Let us consider in more detail the case of $\lambda_1 + \lambda_2 = 0$. Using Volkov's solution [1]-[3] for the electron in the field of the plane wave (40.7), (101.3) and above relations, we obtain the spin structure of the amplitude

$$M = -e_\lambda q \left(\frac{1}{k_1 q} + \frac{1}{k_2 q}\right) \bar{v}^{\delta'}(p') \left\{ \hat{e}_\lambda + \frac{\xi^2 m^2 e_\lambda k}{2(kp)(kp')} \hat{k} - e \left(\frac{\hat{e}_\lambda \hat{k} \hat{A}}{2kp} - \frac{\hat{A} \hat{k} \hat{e}_\lambda}{2kp'} \right) \right\} u^\delta(p) \ , \tag{6}$$

$A = a(l_1 \cos\varphi + \mu l_2 \sin\varphi)$ is the 4-potential of the wave (101.2), μ is the polarization, $\varphi = kx$, $\xi = ea/m$ is the wave intensity, $q^\mu = p^\mu - \frac{\xi^2 m^2}{2kp} k^\mu$ is the quasimomentum (101.4). Making series expansion in Bessel functions $J_n(z)$ (101.7) and integrating in 4-space, we make sure that amplitude Eq.6 of the reaction $sk + q + q' = k_1 + k_2$ completely coincides, to an accuracy of the factor preceding the "sandwich", with the s-channel of single-photon reaction $sk + q = q' + k'$ (101.10). sk denotes coherent absorption from the wave of photons (nonlinearity). The factor itself just reflects that exactly two-photon annihilation takes place.

For laser wave we choose

$$l_1 = \frac{(kv')v - (kv)v'}{\sqrt{(vv')^2 - 1} k_\perp} \ , \ \ l_2 = n_2 \ . \tag{7}$$

Than in (101.6) $\alpha_2 = 0$ and $z = |\alpha_1| = \frac{\xi m |kv'/kv - kv/kv'|}{\sqrt{(vv')^2 - 1} k_\perp}$ is the Bessel function argument. With such a choice in the DSB we have (see Eq.4)

$$e\frac{\hat{e}_\lambda \hat{k} \hat{A}}{2kp} u^\delta(p) = \mu\delta \frac{\xi \hat{e}_\lambda \hat{k}}{2m^2 k_\perp} \sum_{s=-\infty}^{\infty} e^{-is\varphi} \left\{ \left(1+\mu\delta\frac{s_3 k}{vk}\right) J_{s-\mu}(z)\omega_{-\mu} - \left(1-\mu\delta\frac{s_3 k}{vk}\right) J_{s+\mu}(z)\omega_\mu \right\} u^\delta(p) \ . \tag{8}$$

Besides,

$$\hat{e}_\lambda \hat{k} \omega_{-\lambda} = \sqrt{\frac{2k_2 k}{(k_1 k)(k_1 k_2)}} \hat{k}_1 \hat{k} \omega_{-\lambda} \ , \ \ \hat{e}_\lambda \hat{k} \omega_\lambda = \sqrt{\frac{2k_1 k}{(k_2 k)(k_1 k_2)}} \hat{k}_2 \hat{k} \omega_\lambda \ .$$

A similar expression is obtained for the structure $v^{\delta'}(p')\hat{A}\hat{k}\hat{e}_\lambda$. Than rather trivial calculations by formulas , Eqs.1-8 remain.

The obtained formulas permit investigating the spin and nonlinear effects of a number of interesting processes: in the region of conversion (linear colliders); induced decay of ortopositronium with coherent absorption (radiation) of an odd number of photons.

In the t-channel (when $e^\pm \to e^\pm$), this is a double Compton back-scattering (the Compton back-scattering is a basic process in the operation of γ-colliders [7]); 2γ-creation of pairs in the region of conversion (collider testing), etc. Examples of comprehensive studies of single-photon processes are given in [2]-[4], [6].

It should be need that the inclusion of weak interactions does not lead to additional complications in calculations and sometimes even simplifies them. This is connected with the properties of the projection operators ω_λ, namely

$$\omega_\lambda \omega_{\lambda'} = \frac{1+\lambda\lambda'}{2}\omega_\lambda \ , \ \ \omega_\lambda \gamma_5 = \lambda\omega_\lambda \ .$$

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